CHAPTER 19

The Inertial Reference System

ASTRONOMICAL phenomena as directly observed are represented in terms of space and time coordinates in the reference system of time and position defined empirically by a fundamental star system. An immediate observation of the position of a celestial object is a measurement of its apparent position on the sphere relative to selected stars; a determination of time by astronomical observation is a measurement of the position of a local terrestrial meridian plane in its diurnal circuit relative to the stars. The stars directly are the actual reference points; the geometric coordinate systems on the sphere, to which the observations are conventionally referred, are defined by the tabular right ascensions and declinations of the stars from a particular star catalog.

A fundamental star system is intended to represent the reference system defined by the rotational and orbital motions of the Earth. It is established by observing the reflections of these motions in the apparent diurnal motions of the stars and in the apparent motions of the Sun and planets among the stars. The positions of the equator and the equinox among the stars are *empirically* determined from these apparent motions, but by methods designed to trace among the selected standard stars the equator and the ecliptic defined *dynamically* by the theory of the motions of the Earth.

In the mathematical representation of astronomical phenomena constructed from gravitational theory, the motions of celestial bodies are expressed in terms of independent variables derived ultimately from coordinates in an inertial reference frame of position and from a scale of uniform time, and phenomena are represented by relations that conform to the laws of dynamics which implicitly define this reference system of position and time. In principle, therefore, a fundamental star system determines an inertial reference system defined by the motions in the planetary system, specifically by the motions of the Earth; a dynamical frame cannot yet be determined by means of the stars alone, because of very incomplete knowledge of the dynamics of stellar systems.

In practice, however, because of the limitations of the methods available for the construction of a fundamental star system, the reference system which is determined by immediately observed phenomena and empirically established from observations of these phenomena, and which is formally defined by the catalog coordinates of selected stars and their variations with time, cannot actually be assumed with certainty to be rigorously equivalent to an inertial system; essentially, it is only a practical intermediary in the dynamical interpretation of observations which are referred to it.

The object of meridian astronomy is not only to provide conventional empirical reference systems for practical observation, but also continually to improve the positions and proper motions in an effort to eliminate the discordances which still exist between different fundamental star systems, and to determine the exact relation of the reference system of direct observation to the inertial reference system of rational dynamics.

The Reference System of Observation

In determining the relation of a fundamental star system to the inertial system defined dynamically by the motions of the Earth, it is of secondary importance that the star system is inevitably affected by accidental and systematic errors of the observations from which it was derived. For the most part, after the repeated revisions that have been made on the basis of long continued observation, the small errors of this nature which still remain have more the character of practical imperfections than essential defects. The fact which is of primary importance is that a fundamental star system is a *moving reference system*. It is defined by the catalog positions at the epoch, and their *variations* due to *both* precession and proper motion together. These are the quantities that are actually determined by observation in establishing the system; direct observations cannot separate the motions of the stars from the motions of the reference circles. The coordinates of the stars obtained from the catalog represent essentially the positions of the *moving equator and equinox* among a conventionally selected set of *moving stars*, empirically determined by deriving the *relative motions* from repeated observations at successive epochs. The relation between the inertial system and the geometric coordinate system defined by the catalog depends upon the motions of only the reference circles themselves, and cannot be determined until an independent determination of the precession has been made.

An adopted value of the precession is often considered part of the definition of a fundamental system, since it is required in order to determine the proper motions which are commonly included in the catalog. Strictly speaking, however, the precession is not a defining characteristic of the reference system, but rather an extraneous property which defines the relation of the reference system of observation to an inertial system; precession represents a rotational motion of the geometric coordinate system.

It does not explicitly enter into observed positions themselves, but only into their *interpretation.* The coordinates of any celestial object, or the measure of time at any instant, are determined by observations referred directly to the fundamental stars that define the moving empirical reference circles. The immediately observed positions and motions of other stars and of the bodies in the solar system are therefore obtained *directly in the rotating system.* The adopted precession is not ordinarily explicitly used in determining them; if it were, the proper motions of the fundamental stars would have to be combined with it, but the proper motions are by definition the centennial variations minus the precession, and hence in practice the centennial variations are used directly. The coordinates obtained by observation are therefore the same irrespective of the adopted value of precession; the centennial variations cannot be changed except by a revision of the fundamental system on the basis of further fundamental observations.

The dynamical interpretation of the observed planetary and stellar motions, however, is directly dependent upon the value of the precession. In the rotating coordinates defined by the fundamental star system, Newton's laws of motion are not valid. To construct a theoretical representation of the observed motions on the basis of these laws, the reference system of observation must be connected with the inertial system of dynamics, either explicitly or implicitly, by means of a numerical value for the precession; an error in the adopted value is equivalent to a residual rotation of the reference system of the dynamical theory.

In stellar dynamics, the precession is explicitly removed from the centennial variations, in order to obtain the proper motions of the stars, which are the only components of the observed motions that are significant for the dynamics of the stellar system. In the planetary theory, it is not necessary to eliminate the motion of the reference system from the directly observed motions; instead, in practice, it is combined with the theoretical gravitational motions, computed relative to an inertial frame, by means of an abstract mathematical transformation from the inertial system to the moving system of observation. The constants are evaluated directly in the moving system, immediately from observations, without explicitly using the numerical value of the precession; but it necessarily is implicitly involved, and discrepancies may consequently appear between the theoretical motions and the observed motions, equivalent to effects of an error in the adopted value.

A critical coordination of observation and theory, particularly the investigation of discrepancies which cannot reasonably be ascribed to errors of observation or to computational deficiencies of theory, is therefore dependent on an accurate value of the precession.

Since, in principle, the reference circles empirically defined by a fundamental star system represent the equator and ecliptic determined by the rotation and orbital revolution of the Earth, it might be expected that their motions could be calculated from the theory of the motions of the Earth; but in practice only the motion of the ecliptic and the values of the small terms in the theoretical expressions for the motion of the equator can be obtained in this way. The principal parts of the lunisolar precession and nutation must be determined by observation, since the internal structure and constitution of the Earth are not known with sufficient accuracy for a theoretical calculation. The determination of the nutation is facilitated by the periodic character of the observable phenomena which it produces, and moreover a small error is not of great significance in establishing an inertial reference system because of the comparatively short period of the nutation; but peculiar difficulties are encountered in determining the precession separately from the proper motions of the stars, since both are steady progressive motions and only the *relative* motion represented by the centennial variations can be actually observed.

No absolute standard of reference is accessible for overcoming this difficulty. The reference system itself must be established by means of observations on the same objects whose positions relative to this system are to be determined; strictly speaking, no ultimate basis is available on which to interpret with certainty any particular part of the observed motion of the reference circles relative to the stars as being due to a rotation of the coordinate system rather than to a systematic component in the motions of the stars. In ancient times, the observed displacement of the equinox among the stars was attributed entirely to an actual rotation of the *sphere of the fixed stars*. An alternative interpretation, like the interpretation of the diurnal rotation of the stars or the annual motion of the Sun, must be based upon further evidence, additional to the immediate observation of the relative motion alone. From the evidence available, it is probable that the generally adopted value of the precession requires an appreciable correction to eliminate completely the rotational motion of the stellar system as a whole. The uncertainty in the rate of motion of the equinox along the ecliptic is somewhat less than $1''$ per century. The value adopted for the centennial general precession in longitude in the standard system of astronomical constants is $5025''.64$ at 1900, determined by Newcomb in 1897; later determinations by different independent methods have shown that this value is too small, but the corrections range from about $+0''.3$ to $+1''$, and the discordances cannot be explained with certainty.

Meanwhile, on the basis of the accumulated evidence for variations in the rate of rotation of the Earth, the measure of time obtained from this rotation has been recognized to require correction for reducing it to the measure defined by the laws of dynamics. A consistent system of corrections to the adopted precession and the astronomical measure of time which will remove all discrepancies between observation and gravitational theory represents, by definition, a transformation of space and time coordinates from the empirical reference system of observation to the inertial system of dynamics. If not all outstanding discrepancies can be satisfactorily eliminated in this way, they may indicate the possibility of a need for modifications in the law of gravitation or in the laws of motion.

The Reference System of the Planetary Theory

The planetary theories and tables have in general been referred to the moving ecliptic and mean equinox of date, in order that the theoretical positions, when converted to the equatorial system, may represent the same phenomena as observed. Observations, after reduction for nutation, are referred to the equator and mean equinox of date. The moving equinox and equator to which observations are referred are directly defined by the fundamental star system; but the equinox and ecliptic of the tables depend in part implicitly upon the particular methods that were used for constructing the theory and for evaluating the constants and orbital elements from observation. Both in the reference system of the planetary theory which is fixed by the methods of construction, and in the reference system of observation which is empirically determined by the star system, the rate of rotation relative to an inertial system is determined implicitly by the method of establishing the reference system, not explicitly by means of a specific value for the precession; the value implicit in either system must be found afterwards, by an investigation especially for the purpose. Not only may the two reference systems be inexact representations of the moving dynamical frame defined by the motions of the Earth, but moreover their relations to the inertial system may not be rigorously the same; the equinoxes of the two systems are not necessarily identical, either in rate of motion among the stars or in position at the epoch.

The observed positions of the planets from which the orbital elements and other constants are determined are independent of any particular value of the precession; they depend only on the catalog positions and centennial variations of the stars. The theoretical positions depend only on the elements and constants, and are virtually independent of the precession; the sidereal mean motion which is required in developing the theory is obtained from the observed mean tropical motion by subtracting an adopted precession, but otherwise a numerical value for the precession is not explicitly used in the planetary theories. Except for higher order effects from the error in the mean motion which an error in the precession may cause, the tables do not depend upon the value of the precession. However the equivalent is implicitly involved in mathematically connecting the inertial system of the fundamental dynamical equations with the reference system of observation. The particular methods used for developing the theory fix the reference system of the tables and implicitly determine a numerical value for its precessional motion. A discrepancy between this value and the actual dynamical value may appear either in the form of explicit discrepancies between theory and observation, or as errors implicit in the constants that were determined from observations. Errors from this source in the planetary theories may be removed by appropriate corrections derived from an independently determined correction to the adopted precession; or, conversely, it may be possible to obtain a correction to the precession from a comparison of planetary theories with observations.

In the planetary theories, the plane of the ecliptic has in general been used as the fundamental plane of reference. The motion of this plane relative to an inertial system is obtained from gravitational theory, in terms of the planetary masses, and therefore the reference plane is defined dynamically. By reckoning longitudes from a departure point fixed in the moving plane of the ecliptic (e.g., from the fixed equinox of the epoch) a reference system is obtained that is entirely dynamical. The theoretical apparent position of the planet relative to the instantaneous dynamical ecliptic is represented by its geocentric longitude and latitude calculated from the expressions for the heliocentric coordinates, which are functions of the dynamical measure of time, the planetary masses, and the constants of integration. In practice, the numerical values of the constants and parameters are obtained from observation by the method of determining corrections to provisional values by comparing a long series of observations with an ephemeris computed from theory with these provisional values. Corrections to the fundamental star system to which the observations were referred may be determined at the same time, just as corrections to the planetary theories may be obtained during the process of establishing a fundamental star system.

In principle, for the transformations necessary either to convert observed right ascension and declination to longitude and latitude referred to the dynamical reference system, or to convert theoretical longitude and latitude to right ascension and declination referred to the empirical fundamental system, numerical values must be determined for three quantities: the angular distance from the empirical equinox to the intersection of the dynamical ecliptic with the empirical equator, and the inclination of the instantaneous dynamical ecliptic to the empirical equator, in order to fix the position of the theoretical reference plane; and the angular distance between the departure point in the dynamical ecliptic and the intersection of the dynamical ecliptic with the empirical equator, in order to fix the position of the origin from which theoretical longitudes are reckoned. These parameters determine the position of the dynamical reference system of the theory relative to the empirical system of observation, and consequently also the relation of the empirical reference system to an inertial system. The practical procedures that have actually been used to determine them have differed in some details in the different planetary theories which have been constructed from time to time.

Leverrier neglected the possible difference between the empirical equinox and the dynamical equinox. After obtaining the theoretical expressions for the motion of the plane of the ecliptic relative to an inertial system, in terms of the planetary masses,* he equated the resulting theoretical expression for the secular variation of the obliquity, in terms of the masses and the lunisolar precession of the obliquity,† to the numerical value of this variation derived from accumulated observations of the declination of the Sun near the solstices during the preceding hundred years. From this equation of condition among the masses,‡ and further equations of condition obtained from the periodic perturbations of the Earth, the values of the planetary masses were determined.§ These values were then used in the theoretical expression for the variation of the obliquity, to obtain a numerical expression for the obliquity as a function of the time.‖

In the method later used by Newcomb, the assumptions implicit in Leverrier's procedure that the empirical equator and dynamical equator coincide and that the equinox defined empirically by the star system is at the intersection of the equator with the dynamical ecliptic are not made. Newcomb, in transforming ecliptic coordinates to equatorial coordinates, introduced two corrective terms to take account of the angle between the dynamical equator and the equator defined by the star system, and the difference between the empirical equinox and the actual intersection of the dynamical ecliptic and equator. In the comparison of theory with observation, these corrections to the equinox and obliquity, and the secular variations, are determined at the same time as the orbital elements and masses.

These procedures enable the position of the moving fundamental plane of the dynamical reference system to be found relative to the reference system of observation; but the departure point in this plane from which longitudes are theoretically reckoned cannot be directly determined, since its position relative to observable reference points depends upon precession. The quantity that is actually observed is the motion of the planet itself in longitude relative to the moving equinox, which is the sum of the precession of the equinox and the sidereal motion of the planet. In practice, therefore,

* Leverrier, *Ann. de l'Obs. Imp. de Paris* **II**, 103–105.

† Leverrier, *Par. Ann.* **II**, 170–175.

‡ Leverrier, *Par. Ann.* **IV**, 49–52.

§ Leverrier, *Par. Ann.* **IV**, 92–97 and 102.

‖ Leverrier, *Par. Ann.* **IV**, 104.

the planetary tables are based directly on the observed tropical mean motion in orbital longitude; this is the equivalent of using the moving empirical equinox as the cardinal point in the fundamental reference plane instead of the theoretical fixed departure point.

The practical reference system of the planetary theory is consequently partly dynamical and partly empirical. It is established independently of precession by determining from observation a plane that is dynamically defined relative to an inertial system by gravitational theory and a reference point in this plane that is empirically defined by the fundamental star system. The theoretical gravitational motions are expressed in terms of the coordinates in this system, although it is not rigorously the equivalent of the dynamical frame abstractly defined for the planetary theory. The actual relation of the reference system to an inertial system depends upon the precession implicit in its practical construction; this is a potential source of appreciable discrepancies between theory and observation, in addition to errors in the elements and constants and to inequalities in the measure of time.

The Determination of an Inertial System

In the reference systems both of observation and of the planetary theory, the cardinal reference point is the equinox defined by the catalog right ascensions of the fundamental stars. From each of these systems, in principle, an inertial system may be derived by determining the numerical value of the precessional motion of the equinox. In the reference system of observation, by means of a statistical discussion of the observed centennial variations, precession and proper motions may be separated, and an inertial system established empirically. In the reference system of the planetary theory, from a comparison of the observed motions of the planets with gravitational theory, precession may be determined dynamically. In practice, however, serious difficulties are encountered in both methods, and the establishment of an inertial frame of reference is among the most difficult problems of positional astronomy.

The kinematic determination of precession by statistical analyses of stellar motions is strongly liable to systematic error. Any systematic element in the actual proper motions which is not eliminated by the analysis from the observed motions relative to the fundamental system will be incorporated in the value obtained for the precession, and this value cannot be assumed with certainty to be identical with the numerical value of the dynamically defined precession. In effect, the stars, besides being used to define directly the immediate reference system of observation, are also used to define a frame which is pragmatically regarded as inertial; but a determination by an independent method is necessary for an estimate of the possible difference from the actual dynamical frame.

Improved values for precession may be expected in the future from two methods which have not been feasible in the past. A kinematical determination can be made by connecting the fundamental star system with the extragalactic nebulae, after a sufficiently long interval of time for the necessary observations to be completed. Meanwhile, a dynamical determination of precession may be based upon the variations of the nodes and perihelia of the planetary orbits after some improvements in the planetary theories, particularly in the values of the masses of the planets.

Dynamical Determination of Precession

A determination of precession from the motions in the solar system depends principally upon second-order effects of the precession on planetary motions, since the quantity that is affected to the first order is the mean motion which is one of the constants of integration. The sidereal motion, which is obtained in practice by subtracting an adopted precession from the observed tropical motion, is connected with the mean distance a by the dynamical relation $n^2a^3 =$ constant that defines a; from a direct measurement of a in astronomical units, the value of n could be derived and used to determine precession from the first-order effect.

The possibility in principle of establishing an inertial system from observations of dynamical phenomena in the solar system depends upon the fact that, because of the precessional rotation of the reference system of observation, the directly observed motions of the planets are in part the result of accelerations not produced by the action of forces. If the gravitational forces that are acting can be determined with precision and certainty, the rotation of the reference system can be derived from the differences between the actual accelerations relative to this noninertial frame, and the accelerations that would occur from the action of the forces in accordance with the laws of motion were the frame inertial. However, an effective application which will lead to conclusive results requires improved values of the masses of the planets.

The effects of a rotation of the reference system on the apparent motion of a planet may be represented in the form of expressions for variations of the orbital elements, particularly the secular motions of the perihelion and node, in terms of the angular velocity of the system. The observed values of these secular variations, like the observed mean motion of the planet, are referred to the moving equinox and are therefore affected by precession; but the sidereal components, unlike the sidereal component of the mean motion, may be computed from gravitational theory. The theoretically calculated motions of the planetary perihelia and nodes produced by the action of forces, including relativity effects, are the values relative to an inertial frame; comparing them with the observed motions relative to the moving equinox defined by the star system is a means of determining the precession, since the differences represent the variations due to the rotation of the reference frame.

However, the perihelia and nodes are not themselves observable. Their positions and motions in space must be inferred from observations of the apparent positions of the planets extending over long intervals of time; and the comparison with the theoretical motions, from which in principle the precession may be obtained and a dynamical reference system established, is indirectly made by analyzing the discrepancies $O—C$ between the observed and theoretical positions of the planets. For example, the principal term of the equation of the center in the orbital longitude is $2e \sin(L - \pi)$, in which L is the mean longitude from the equinox of date and π is the longitude of the perihelion from this equinox. An error $\Delta\pi$ in the adopted value of π will introduce into the tabular longitude of the planet a periodic error $-2e\, \Delta\pi \cos(L - \pi)$ which will appear as a discrepancy between the observed and the computed longitudes and may be determined by an appropriate analysis of the differences $O—C$. Since $\pi = \pi_0 + bt$, where b is the secular motion, and $\Delta\pi = \Delta\pi_0 + (\Delta b)t$, the residuals $O—C$ will contain a term with the period of the revolution of the planet and amplitude increasing proportionally to the time; the coefficient Δb may be interpreted as representing the error in the value of the precession that was used to obtain b in determining the tabular longitude of perihelion.

Similarly, the secular motions of the nodes may be used. In principle, the rotation of the reference system may be determined from the discordances between theory and observation by means of a least square solution for the components of this rotation around the line of equinoxes, the perpendicular to this line in the plane of the ecliptic, and the normal to the plane of the ecliptic. Attempts were made by Anding in 1905 and by others early in the twentieth century to apply this principle to determine a correction to the precession from the differences found by Newcomb* between the observed secular variations of the orbital elements of the four inner planets and the theoretical values computed from masses determined independently of these variations; but these attempts were before the corrections required by the theory of relativity were known, and they led to inconclusive or inadmissible results. Later, Bauschinger† made another attempt, with somewhat more success. However, in practice, no conclusive correction to the precession can yet be obtained by this method. The discrepancies between theory and observation are caused not only by the systematic rotational motion of the

* S. Newcomb, "Astronomical Constants," pp. 109–110, 1895.

† J. Bauschinger, *Naturwissenschaften* **10**, 1005–1010 (1922).

star system that may be erroneously incorporated in the precession but also by errors of observation and by deficiencies in the gravitational theories, especially from uncertainties in the planetary masses.

Empirical Determination of Precession

The methods that have been used for the empirical determination of precession depend upon the discussion of the observed centennial variations of a very large number of stars by means of a statistical treatment based on some hypothesis about the general character of the motions of these stars as a whole. Early determinations of the precessional constant were usually based on the hypothesis that the motions of the stars in space are at random so that any systematic component in the observed centennial variations, common tc the stars as a whole, is due to precession and to the effects of the motion of the Sun relative to the stars; but this hypothesis has been invalidated by the discovery of galactic rotation and other systematic stellar motions. A determination of precession from the observed motions of the stars by means of the classical kinematic methods requires empirical hypotheses concerning (1) the general structure and motion of the stellar system as a whole, (2) the progressive motion of the Sun through space, and (3) the peculiar motions of the individual stars. The determination of the constant of general precession with high accuracy is exceedingly difficult because of the difficulty of eliminating the systematic effects of the solar motion, star streaming, galactic rotation, and systematic errors in the observed positions. The value for the precession depends upon the particular hypotheses that are adopted and upon the particular stars that are used for the determination.

The usual method of determining precession is to derive a correction to an adopted value by analyzing the proper motions of a large number of stars distributed over the entire celestial sphere. The proper motions of the stars are obtained by calculating the general precessions in right ascension and declination with an adopted value of the constant of precession, and subtracting them from the centennial variations relative to an adopted equinox and declination system. Consequently, the proper motions as thus obtained in practice are the variations of right ascension and declination resulting from: (1) a remaining precessional motion of the equinox relative to the inertial system, due to the error in the adopted precession, (2) the parallactic motions of the stars caused by the motion of the Sun, (3) the systematic motions due to the rotation of the stellar system as a whole, and (4) the further systematic and random motions through space peculiar to the individual stars.

In addition, the values obtained for the proper motions include apparent variations due to systematic errors of observation in the star system. Theoretical expressions may be derived for the variations of right ascension and declination produced by the error in the precession, the parallactic motions, and the galactic rotation. On the hypothesis that the peculiar motions are sufficiently at random to be considered in the aggregate as of the same nature as accidental errors, these expressions may be equated to the numerical values of the proper motions, and the resulting system of equations solved for the separate components by the method of least squares.

The general precessions in right ascension and declination are

$$P_\alpha = m + n \sin \alpha \tan \delta,$$
$$P_\delta = n \cos \alpha,$$

where

$$m = \psi \cos \epsilon - \lambda,$$
$$n = \psi \sin \epsilon,$$

and ψ denotes the lunisolar precession in longitude, λ the planetary precession, ϵ the obliquity. The variations of right ascension and declination due to the corrections $\Delta\psi$ and $\Delta\lambda$ or Δm and Δn required to the precessional motions are expressed by the partial derivatives of P_α and P_δ with respect to ψ and λ,

$$\begin{aligned} \cos \delta(\Delta\alpha) &= (\cos \epsilon \cos \delta + \sin \epsilon \sin \alpha \sin \delta)\, \Delta\psi - \cos \delta\, \Delta\lambda \\ &= \cos \delta(\Delta m - \Delta\lambda) + \sin \alpha \sin \delta\, \Delta n, \\ (\Delta\delta) &= \sin \epsilon \cos \alpha\, \Delta\psi \\ &= \cos \alpha\, \Delta n. \end{aligned}$$

Thus, from the declinations the value of n, the annual motion of the celestial pole toward the equinox, may be determined; and from the right ascensions, the value of m, the annual motion of the equinox along the equator.

By means of a least square solution of the equations of condition formed by combining these expressions with those for the effects of solar motion and rotation of the galaxy, the correction to the adopted precession may be determined, together with the solar apex and the constants of galactic rotation. However, the errors of observation in the centennial variations may introduce systematic effects into the proper motions, and these effects will be incorporated in the results of this analysis of the motions. In particular, the proper motions in right ascension may have a constant error $-\Delta e$, due to changes in observational techniques or to other observational and computational effects. For this reason, in the above equation for $\Delta\alpha$ the coefficient of $\cos \delta$ on the right that is to be determined is usually taken to be

$$k = \Delta m - \Delta e - \Delta\lambda.$$

The correction Δe is the mean value of the proper motions in right ascension after other corrections have been applied; it is commonly called the *motion of the equinox*, but this name is somewhat ambiguous, since a correction of the same form may also be obtained in other ways, and the interpretation of the value obtained may not be the same.

For example, from a discussion of meridian observations, a correction to the equinox may be determined in the form $a + bT$, where a is a correction to the zero point of the catalog (i.e., a constant correction to all the catalog right ascensions) and b is the centennial variation of the correction (i.e., a constant correction to the centennial variations in right ascension). The term bT must not be confused with precession. The correction b to the centennial variations has also been called the *motion of the equinox*, but it does not necessarily have the same value as the correction Δe obtained from an analysis of proper motions. The value of Δe depends upon the methods and hypotheses by which the proper motions were obtained from the centennial variations and analyzed into components.

The inherent difficulty in empirically establishing an inertial system by the usual statistical methods is the dependence of the results on the particular hypotheses upon which the equations of condition are based, and on the particular observational material that is used. There is no certainty that the various systematic effects have been completely separated. As a means of avoiding these defects, the use of the extragalactic systems as a reference frame in the future is planned. The velocities of these systems across the line of sight are unknown, but even though they may be of the same order as the velocities of recession the distances are so great that the angular motions are virtually zero, and the apparent positions of these objects may be considered for all practical purposes to provide an inertial frame of reference. If the positions of a large number of the distant galaxies were determined relative to the fundamental stars, we should have a means not only of correcting and controlling the fundamental star system but also of determining the constant of precession more precisely. A proper separation of the centennial variations into the component due to precession and the part due to actual motions of the stars would then be possible.

The project to use the external galaxies as an inertial reference system was initiated by W. H. Wright.* Observations were commenced at the Lick Observatory in 1947 by C. D. Shane. A series of photographic plates covering the part of the sky visible from the Lick Observatory was completed in 1954. It is expected that the future second-epoch plates will be completed about 1985. The principal practical difficulty is in devising procedures of sufficient accuracy for connecting objects as faint as the extragalactic systems with the comparatively bright stars that have been observed with transit circles. In principle, the connection could be established directly by

* See W. H. Wright, *Proc. Amer. Phil. Soc.* **94**, 1–12 (1950).

photography of the position of the celestial pole* as well as by a photographic survey of the entire sky; but it is difficult to construct a sufficiently stable instrument for this purpose.

The System of Astronomical Constants

In addition to the precession, a large number of other constants and numerical parameters are involved in the astronomical reference systems of time and position. Their numerical values are essential to the practical realization of the reference systems, and to the theoretical representation of the positions and motions of the celestial bodies. These constants and parameters may therefore be considered to constitute a part of the space-time frame formed by the astronomical reference systems collectively. The system of constants, the planetary tables, and the fundamental star system cannot properly be considered independently of one another. Strictly speaking, all three should constitute a homogeneous consistent system, but this ideal has not yet been attained.

Numerical values for the fundamental astronomical constants and the principal elements of the planetary system may in general be obtained from observation in several different ways, often by two or more completely independent methods. Inevitably, different determinations of the same constant, either by different methods or at different times, will give somewhat different values. The discordances are merely a reflection of the elementary principle of physical science that no measurement can be perfect and that different determinations may reasonably be expected to differ.

The methods that have been used for determining the values of astronomical quantities by observation are of two types; (1) direct measurements of individual quantities separately, each by means of a few special observations expressly designed for the specific determination of this particular quantity; (2) indirect determinations, usually in the form of corrections to provisionally adopted values, by means of an adjustment of a large accumulation of routine general observations that have been carried on over an extended period of time. The discussion and adjustment of these accumulated observations will in general enable several different quantities to be evaluated together.

After a fairly accurate value of a quantity has become known, a redetermination by the second method is the most satisfactory and advantageous way of obtaining an improved value, and is the procedure now generally followed; but until comparatively recent times during the historical development of astronomy, the first method was necessarily the predominating practice. Reasonably good values of most of the principal constants and fundamental

* See H. N. Russell, *Mon. Not. Roy. Astr. Soc.* **73**, 735–742 (1913).

parameters may be obtained by very simple means, and have been known since ancient times. From successive determinations during past centuries, the values of the principal astronomical constants are now known far more accurately than they can be measured by any single special observational determination; any further increase in accuracy by means of observational measurements can be obtained only by the method of determining, not the quantity directly, but a correction to a previous value. The construction of a precise system is a process of successive approximation, which depends upon continued observation over long intervals of time, and periodic discussions of the accumulated data.

However, from accurate values that have been determined for each of a number of different quantities from observation, still better values may often be obtained by adjusting the measured values to conform to theoretical relations that are known to connect the true values. Many such relations exist among the fundamental constants of astronomy; but the measured values usually do not exactly satisfy these relations, and hence are not entirely consistent with one another. The standard system of fundamental constants adopted internationally for use in astronomical ephemerides is in part based on an adjustment of observed values; but it has not yet been possible to construct a system that is perfectly consistent and in which, at the same time, the values of all the quantities conform to the best available measured values to within the uncertainty of the observations.

An individual determination of any constant, even though it be the most recent measurement and made with improved instruments and methods, is of relatively little significance by itself. The true value is unobtainable with certainty; any individual determination gives only a range of values within which it is more or less probable that the true value lies. The differences between different determinations, especially when by independent methods, are often greater than can be explained by the known sources of accidental and systematic errors to which these determinations are liable. No particular one can be regarded as definitive; only by a critical comparison and discussion of all the accumulated determinations that have been made may a value be obtained that can be considered the most probable or most reliable at the present time.

Moreover, the particular value of a constant which it is most advisable to adopt depends upon the purpose for which it is to be used. It is often desirable to adopt different values of the same quantity for different purposes. Precise ephemerides of the Sun, Moon, planets, and principal stars are primarily for comparison with accurate observations in order to improve the theories and tables from which these ephemerides are computed. For this purpose it is much more important that a standard system of values be conventionally adopted and consistently used, than that changes be continually made in an effort to conform to what may appear at the moment to be the best available value of each quantity. Since it is not to be supposed that any of the adopted values are exactly correct, we may expect to find small discordances between the observed positions of the celestial bodies and the positions computed with the adopted constants. These discordances are one of the most valuable means of determining probable corrections to the adopted values. Endless difficulty, labor, and liability to error would result from frequent or unsystematic changes in the bases of the ephemerides. The general practice is to continue to use the same constants and tables until long series of observations at many different observatories shall unmistakably show significant corrections to be necessary, and even then to make no changes until a systematic revision of the whole system can be made and universally adopted.

The constants listed in the national ephemerides are not intended to be either the "best" or the most recently determined values as is sometimes mistakenly supposed, but to show the standard system of values used, by international agreement among astronomers, for the particular purpose of computing fundamental ephemerides. In a special computation for some other purpose, there is sometimes reason to use the most reliable value of some quantity that is available at the time rather than the conventional standard value; but only at long intervals is it advisable or even practicable to revise the standard system of constants from which the fundamental ephemerides are computed.