THE EXPERIMENTAL AND ANALYTICAL PROGRAMS FOR RE-ENTRY BURNUP OF SNAP REACTORS*

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ABSTRACT

The use of nuclear power in space applications is complicated by the radiological hazards that would develop if a radioactive component of a nuclear system should re-enter the earth's atmosphere, strike the earth, and contaminate a portion of terrestrial surface. Because of these hazards, analytical and experimental programs are under way to study the possibilities of re-entry burnup of these nuclear systems. The results of the preliminary analysis have shown that complete re-entry meltdown, burnup, and subsequent dispersion of radioactive material would take place for the SNAP Z fuel elements above 200,000 feet.

To facilitate analysis, step changes were postulated to occur in the equivalent radius of the re-entrant body to simulate the changing geometries as the system breaks up. The reactor and components were assumed to have a spherical geometry. With these assumptions the altitude for complete meltdown of the different components was calculated. The model with spherical symmetry was used, except for the actual SNAP 2 fuel rods which were treated individually as tumbling cylinders. A constant re-entry angle of one degree was assumed for the calculations (to approximate a decaying orbit), and each component was assumed to be completely melted without any vaporization before the next component was exposed to heating. The atmospheric density was described by:

\[ \rho = \rho_0 e^{-\beta \gamma} \]

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A similar analytical study was made for the SNAP 10 system. Results of this study predicted complete burnup for SNAP 10 fuel plates above 190,000 feet.

The first phase of the experimental program consisted of heat transfer studies in which SNAP 2 fuel rods were subjected to simulated re-entry conditions in a continuous arc-jet. Excellent agreement was found between the experimental and analytical results and deviations between analysis and experiment were no greater than 10%; the analytical and measured ablation rates agreed within 10%. This agreement would tend also to verify the assumption of minimal vaporization.

I. INTRODUCTION

This paper presents the current analytical and experimental studies of the SNAP re-entry hazards program. The purpose of this program is to determine the effects of atmospheric re-entry on SNAP systems, project design criteria to facilitate adequate atmospheric burnup and dispersal of the radioactive materials, and to estimate the radiological hazards resulting from re-entry (Ref. 1).

II. ANALYTICAL PROCEDURE

A. RESULTS

1. SNAP 2 Re-entry

The re-entering reactor was assumed to re-enter enclosed in the satellite vehicle. When the total heat absorption and temperature is sufficient to completely melt the vehicle and radiator, these will be swept away with the radiation shield leaving the reactor and its reflector exposed to the atmosphere. Each reactor component will be successively swept away due to the melting of the component, or in the case of the reflector, the reflector will be swept away as the fasteners are melted. At the point of fuel rod release, the fuel rod burnup was considered for an individual fuel rod. The above procedures were employed as it would appear that these would be the logical succession of actual re-entry events that would occur. The altitude and velocity predicted for these re-entry events are depicted graphically in Figures 1 and 2.
and pictorially in Figure 3. It was predicted that complete meltdown of the SNAP 2 fuel rods would take place above 200,000 feet (Ref. 2).

2. SNAP 10 Re-entry

The analytical investigation indicated that complete burnup for SNAP 10 will occur above 190,000 feet for a re-entry angle of one degree. The re-entry events are shown graphically in Figure 4. It appears that no splitting of the fuel plates will be necessary to insure burnup as adequate heat is available to melt each fuel plate upon re-entry.

Only about one-half the volume of each of the beryllium reflector and conductor blocks will melt upon re-entry due to the high heat of fusion and partial sublimation of the beryllium. These are not highly radioactive. As in the case of the fuel plates, the beryllium plates and blocks were treated individually. Reduction of size or splitting of these beryllium pieces does not significantly help the re-entry burnup of the beryllium. It may be seen from Figure 4 that the temperatures reached by the re-entrant body are adequate to melt the various components (Ref. 3).

B. ANALYSIS AND DEVELOPMENT OF EQUATIONS

It was first necessary to describe the effect of atmospheric drag on the re-entrant body. It was determined that the friction drag equation would adequately describe the effect of atmospheric drag, and the change in altitude due to velocity and re-entry angle was also stated. Next, the heating due to atmospheric friction was described. The stagnation heat input was averaged over the total area of the body by a modifying factor as described in the development of the equations.

The integrated form of the heating equation, Equation 14, was obtained by analytical integration of the combination of the drag, altitude, and heating equations. The analysis treats the burnup of the individual components as the vehicle enters the atmosphere; after the reactor vessel is melted, the rods are treated individually.

The equilibrium temperature reached by each re-entrant component was calculated by use of the stagnation heating equation, and the result is given in Equation 15. The following assumptions were used in the analysis:

1) The radius of the re-entrant vehicle is assumed to be constant.

2) It is assumed that due to local burning, cavities will form causing the body to rotate during re-entry.
3) It was also assumed that fluid formed by melting is immediately swept away, i.e., the heat of ablation is equal to the sensible heat plus the heat of fusion. This neglects run-back considerations.

4) The re-entry angle, $\theta$, is assumed to be constant over the range of heating and equal to one degree. This is acceptable because the velocity vector, due to gravity, is small in comparison to the overall velocity.

5) The thermal conductivity of the components is sufficiently large to ensure uniform temperature throughout the component.

6) It is further assumed that the average equilibrium temperature at the surface of the re-entrant body is specified by Equation 15. Use of this equation results in a conservative estimate as it does not take into account the heat radiated to the body from the hot gas cap over the front half of the re-entrant body.

7) No oxidation takes place during re-entry. This, of course, is a conservative assumption as the possible oxidation reactions are exothermic and may occur. The problem of oxidation constitutes a further study.

8) The re-entry body is assumed to be a sphere with a volume equal to that of the actual re-entrant body. This is a conservative assumption since such a sphere presents less area for heat transfer than the actual component presents.

The heats for burnup or meltdown were calculated from the heat capacities and latent heats of fusion for the vehicle and radiator, stainless steel can, stainless steel straps, and the reactor vessel and the individual rods. The meltdown heat for the rods included the heat needed to vaporize the hydrogen. The meltdown heats were substituted in Equation 14 and the complete re-entry meltdown altitude for each portion of the vehicle and reactor was calculated by step changes in the meltdown heats, radius, and body density. The atmospheric density may be described by $\rho = \rho_0 e^{-\beta y}$. This equation assumes an isothermal atmosphere and shows a maximum deviation of about 20% above actual density at sea level which is not important at the altitudes at which this analysis takes place. For either ballistic or satellite re-entry, gravity effects may be neglected (Ref. 4), and the angle of entry, $\theta_E$ may be
assumed constant. The drag is expressed by Equation 2, and the drag coefficient is assumed to be constant and equal to 4/3. The assumption of a constant drag coefficient of 4/3 is acceptable over the Mach number range covered, as this drag coefficient, due to the shock wave, does not vary because of the invariant configuration of the shock wave at these velocities.

For altitude change

\[ dy = U \sin \theta \, dt \]  

(1)

The velocity change for a sphere due to drag

\[ M \dot{U} = \frac{4}{3} \pi r^3 \rho B \dot{U} = -AC_D \rho A \frac{U^2}{2} \]  

(2)

or

\[ \dot{U} = -\frac{\rho A}{\rho B} \frac{U^2}{2r} \text{ if } C_D = 4/3 \]  

(3)

Combining Equations 1 and 3 and integrating

\[ U = U_E e^{-\beta y} \]  

(4)

where

\[ C_2 = \frac{\rho_o}{\rho B \frac{U^2}{2r} \sin \theta} \]

based on the assumption that the atmospheric density may be described by \( \rho = \rho_o e^{-\beta y} \), where \( \rho_o = 0.1095 \text{ lb/ft}^3 \) and

\( \beta = 1/22,000 \text{ feet (Ref. 5)} \).

The heat input mechanism at the stagnation point may be described by Equation 5.

This assumes:

1) Laminar flow, which is the case for flow behind a shock wave.
2) No re-radiation, which is acceptable for the low melting point materials in question, and is partially balanced by neglecting the radiation heat input from the hot gas cap.

3) Constant Prandtl number is assumed because of the almost invariant Prandtl number of air at high temperatures.

The equation for aerodynamic heating at the stagnation point is

$$\dot{q} = \frac{N}{\sqrt{T}} \sqrt{\frac{\rho}{\rho_0}} \left( \frac{U}{U_E} \right)^{3.25}$$  \hspace{1cm} (5)

where $N = 20,800$ (Ref. 6).

A factor, $\eta$, may be used to modify the stagnation heating to the overall heating, i.e., $\eta = 1/4$ (Ref. 7)

$$\int \dot{q} A \, dt = Q_t,$$  \hspace{1cm} (6)

or

$$Q_t = \int_{t_1}^{t_2} \frac{NA}{\sqrt{r}} \sqrt{\frac{\rho}{\rho_0}} \left( \frac{U}{U_E} \right)^{3.25} \, dt$$

where $A = 4\pi r^2$.

$$Q_t = C_3 \int_{t_1}^{t_2} \frac{1}{2} \beta y \, 3.25 C_2 \, e^{-\beta y} \, dt$$  \hspace{1cm} (7)

where $C_3 = Nr^{1.5} \pi$.

From Equation 1, $dt = \frac{dy}{U \sin \theta_e} = \frac{dy}{U_E e^{-\beta y} \sin \theta_e}$.
Substituting in Equation 7

\[ Q_t = C_4 \int_{y_1}^{y_2} \frac{1}{2} \beta y \ e^{2.25 C_2 e^{-\beta y}} \ dy \]  

where

\[ C_4 = \frac{N_r \frac{1.5}{\pi}}{U_E \sin \theta_E} \]

Let

\[ e^{\frac{1}{2} \beta y} = x \]

\[ dx = e^{\frac{1}{2} \beta y} \ dy. \]

Substituting in Equation 8

\[ Q_t = C_5 \int_{x_1}^{x_2} \frac{C_6 x^2}{x} \ e^{x \ e^{-x \ e^{\frac{1}{2} \beta y}}} \ dx \]

where

\[ C_5 = - \frac{N_r \frac{1.5}{\pi}}{U_E \sin \theta_E \beta} \]

\[ C_6 = \frac{\rho_o (2.25)}{\rho_B 2 \pi r \sin \theta_E \beta} \]

then

\[ Q_t = C_5 \int_{x_1}^{x_2} e^{C_6 x^2} \ dx \]
Let

\[ Z = \sqrt{-C_6} x \]

\[ dZ = \sqrt{-C_6} \, dx \]

Substituting

\[ Q_t = \frac{C_5}{\sqrt{-C_6}} \int_{x_1}^{x_2} \sqrt{-C_6} = Z_2 \quad e^{-Z^2} \, dZ \quad \text{(11)} \]

\[ Q_t = \frac{C_5}{\sqrt{-C_6}} \sqrt{\pi} \frac{\sqrt{\pi}}{2} \text{erf} \, Z \quad \text{(12)} \]

where

\[ Z = \sqrt{-C_6} x = \sqrt{-C_6} \, e^{\frac{1}{2} \beta y} \]

\[ \sqrt{-\frac{\rho_0 (2.25)}{\rho_B 2 \sin \theta_E \beta}} \, e^{-\frac{1}{2} \beta y} \quad \text{(13)} \]

Then

\[ Q_t = \frac{C_7}{\sqrt{-C_6}} \left( \text{erf} \, \sqrt{-C_6} \, e^{\frac{1}{2} \beta y_2} - \text{erf} \, \sqrt{-C_6} \, e^{\frac{1}{2} \beta y_1} \right) \quad \text{(14)} \]

where

\[ C_7 = -\frac{N_r}{U_E \sin \theta_E \beta} \]

The average equilibrium temperature reached by the re-entrant body on the surface may be expressed by
\[ \epsilon \sigma T^4 = \dot{q} \eta, \quad (15) \]

where \( T \) is the temperature of the surface radiating into space and \( \dot{q} \) is described by Equation 5.

C. CONCLUSIONS

The analytical study has shown that satisfactory re-entry burnup will take place for both the postulated SNAP 2 and SNAP 10 systems. It should be emphasized that complete re-entry destruction of the SNAP units and subsequent dispersal of the radioactive material depends greatly on the physical design of these units. A system of fuseable links will be necessary to ensure destruction of the integrity of the auxiliary power system, and to release beryllium reflectors as well as the neutron shield at an altitude and velocity that will allow complete burnup of the fuel material. If the shield or reflector remains attached to the reactor, the large heat capacities of these might prevent fuel rod burnup.

III. THE EXPERIMENTAL PROGRAM

A. INTRODUCTION

During January 1960, simulated arc-jet re-entry tests on SNAP 2 fuel elements were conducted. The results of the Phase I study are reported in Ref. 8. The Phase I tests consisted of ablation studies at four re-entry conditions; the samples were mounted such that the stagnation heating was either axial or radial. The four re-entry tests were conducted with air at simulated altitudes and velocities of (1) 250,000 ft and 24,000 ft/sec; (2) 220,000 ft and 21,000 ft/sec; (3) 200,000 ft and 18,000 ft/sec; and (4) 175,000 ft and 13,000 ft/sec. Point 1 closely approximated the actual re-entry conditions that will be experienced by SNAP 2 upon re-entry.

B. RESULTS

Excellent agreement with theory was obtained between the Plasmadyne results and the theoretical evaluation (Ref. 8). The data correlate, within 10% agreement, was obtained for mass loss rate vs stagnation heat rate based on the measured and theoretical ablation rates as seen in Figure 5. In the case mentioned above, the simulated tests yielded higher ablation rates and correspondingly lower total melt-down times due, probably, to the fact that all the material swept off the test models...
was not completely melted, whereas the theoretical model assumed that all material lost was melted prior to being swept off. This also supported the assumed heat of ablation.

G. TEST PROCEDURE

The arc-jet was first calibrated to match the four required flight conditions. This was done by regulating the tunnel jet's mass flow rates and power levels so that the correct pitot pressure and stagnation enthalpy were obtained. These parameters were chosen rather than the heat transfer rate as there is more uncertainty about the heat transfer rate at these high altitudes than the flight parameters chosen.

After calibration was accomplished, several test runs were made in order to eliminate any experimental difficulties that might crop up during the actual experimental runs. After the test runs had been conducted, at the four re-entry conditions, the experimental runs were started.

The test runs consisted of starting the jet, checking the calibration point, measuring the gas parameters, and inserting the sample into the gas stream.

Measurements were made during the period when the model was in the jet; these were, time to heat up, time to ablate, length of run, and temperature of the model surface. The models were weighed before and after each run and the mass change and length change were recorded. From these data the mean loss rates and the stagnation point ablation rates were calculated. Three runs apiece were made for Points 1 and 2 in both axial and radial tests, and two runs apiece were made for Points 3 and 4 for axial tests; with one run at Point 3, and none at Point 4 for the radial tests. Fastax and/or slow motion pictures were taken during each run.

D. ANALYSIS OF TEST RESULTS AND CALCULATIONS

Each test yielded results that were within 10% of theory except one run in the axial tests at Point 1 and the various points at 4. The latter was not expected to correlate due to the fact that the melting point of the ZrH would not be reached at this low energy point in the jet. The former difficulty at Point 1 may be due to a bad sample as this sample exploded rather than melted (Ref. 8).

The radial results gave a good check of the axial results, but the radial results were not used in the calculation of the points in Figure 5, as there is no method to estimate the heat losses along the length of the models. This is due to the fact that the jet sprays out along the length of the model during testing because the model is longer than the jet diameter.
The Phase II tests consisted of several tests at the same re-entry conditions, using a tumbling model, the length of which is not longer than the jet diameter. Tests were made with air and nitrogen to determine the oxidation effects.

Figure 5 was produced by taking the measured ablation rates and reducing these to mass loss rates and comparing these to the theoretical mass loss rate for a given stagnation point heat transfer rate. Points 1 through 4 were chosen for simplicity and compared to the theoretical re-entry cases.

NOMENCLATURE

\[ A = \text{Area ft}^2 \]
\[ C_D = \text{Drag coefficient} \]
\[ C_P = \text{Heat capacity - Btu/}^\circ\text{F-lb} \]
\[ M = \text{Mass -lb} \]
\[ N = \text{Heating coefficient - 20,000 Btu/sec-ft}^{1.5} \]
\[ Q_t = \text{Heat to melt - Btu} \]
\[ q = \text{Instantaneous stagnation heating Btu/sec-ft}^2 \]
\[ r = \text{Radius - ft} \]
\[ t = \text{Time-sec} \]
\[ T = \text{Temperature - } ^\circ\text{R} \]
\[ U = \text{Velocity - ft/sec} \]
\[ y = \text{Altitude - ft} \]
\[ \beta = \text{Constant - 1/22,000 ft} \]
\[ \eta = \text{Overall heating factor - 1/4} \]
\[ \epsilon = \text{Emissivity} \]
\[ \theta = \text{Re-entry angle - degrees of arc} \]
\[ \lambda = \text{Latent heat - Btu/lb} \]
\[ \rho = \text{Density - lb/m}^3 \]
\[ \rho_o = 0.1095 - \text{lb/m}^3 \]
\[ \sigma = \text{Stefan-Boltzman constant} = 4.80 \times 10^{-13} \text{Btu/ft}^2\text{-sec}(^\circ\text{R})^4 \]

Subscripts:

B - Re-entrant body
E - Refers to the point of re-entry of any particular component
t - Total
REFERENCES


2. P. D. Cohn, "The Preliminary Investigation of SNAP 2 Re-entry Burnup," NAA-SR-4890 (February 12, 1960) (S)

3. P. D. Cohn, "The Preliminary Investigation of SNAP 10 Re-entry Burn-up," NAA-SR-MEMO-5036 (March 2, 1960) (S)


Fig. 1. SNAP 2 re-entry burnup procedure.

Fig. 2. SNAP 2 re-entry burnup sequence.
Fig. 3. SNAP 2 re-entry sequence.
Fig. 4. SNAP 10 re-entry sequence.

Fig. 5. Comparison of measured mass loss rate to theory.