ENERGY STORAGE IN SUPERCONDUCTING MAGNETIC COILS

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Abstract

An inductance coil made of superconducting wire and maintained at the temperature of liquid helium is considered as a means of electrical energy storage. A method is described for obtaining an effective d.c. output from discharge of such a coil. Optimization of the coil design for maximum energy storage yields energy per unit volume figures in the range of 2×10^8 joule/m^3 for small coils producing the lower magnetic fields to 2×10^9 joule/m^3 for very large coils producing the higher magnetic fields. The energy per unit weight is comparable to or better than that of the best batteries. The chief advantage of inductive energy storage is the capability of providing a power output greatly exceeding that of a battery of the same weight.

Introduction

With the recent development of high field superconducting wire, the long-term storage of energy within a high inductance superconducting coil becomes a promising possibility. For many years large inductance magnetic coils have been used in laboratories in experiments requiring high energy delivery rates, up to 500,000 amp for a period of msec.[1] Because of resistive losses in the coils, a large amount of power was required to maintain the current through the coil prior to discharge through the load. During discharge, the output current from the coil would decrease exponentially from its initial steady-state value to zero, assuming that the load resistance were sufficiently high to prevent current reversal.

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By using "hard" superconductors which are capable of generating very high magnetic fields, it is possible to store energy in the magnetic field created by the coil without additional expenditure of energy, once the current through the coil is established. After such a coil is energized and its input leads short-circuited, the current will continue to flow without diminution for an indefinite period of time, as there are no resistive losses. At any given time the energy stored in the coil may be withdrawn through a load switched into the circuit.

A superconducting energy storage unit has the serious disadvantage of requiring liquid helium for its operation to maintain its temperature in the vicinity of 4°K. Another problem is the exponentially decaying current, characteristic of the discharging inductance coil, which cannot be utilized easily for powering electrical equipment. A solution to this problem is presented in the present paper. Furthermore, a rapid calculation of the energy storage capacity of a superconducting solenoid of conventional geometry may show that it has a storage capacity per unit weight much less than that of the battery. By optimizing geometry, however, it will be seen that the superconducting coil can provide an energy-per-unit-weight figure comparable to that of the best batteries. Its superiority over the battery is its ability to deliver power to a load at a level several orders of magnitude higher than is possible for the battery. Battery applications in general are limited to power requirements in the tens of kilowatts. On the other hand, the power output of a small, properly designed, superconducting coil can range from megawatts for a period of seconds to kilowatts for many minutes.

Properties of Superconducting Wire

There are now available two types of superconducting wire suitable for producing high field coils. One consists of three parts niobium to one part tin, which is packed as a powder into a niobium tube, drawn into a fine wire, wound into the desired coil configuration, and then reacted in an oven to form the superconducting Nb_3Sn intermetallic alloy. After reacting, this material becomes very brittle. The other material, which is in somewhat wider use, is the alloy Nb-25 at. % (atomic percent) Zr, which has both high ductility and strength.

The current capacity of a superconductor is a function of the applied magnetic field. There exists a critical field above which the material loses its superconductivity. This critical field is determined by the temperature of operation and the degree of cold working of the material. A convenient temperature for operation is that of liquid helium at atmospheric pressure $4.2^{\circ}K$. The influence of cold working dictates the use of very fine wires for maximum current density. The variation of current density with applied magnetic field for the two materials is shown in Fig. 1.[2-3] The Nb_3Sn wire appears to be greatly superior to the other. The curve is somewhat misleading, however, because the indicated current density J is that in the core material alone, and the area occupied by the non-superconducting niobium jacket is neglected.

Design of the Inductive Storage Device

The general design features of the proposed inductive coil for energy storage† are shown in Fig. 2. The coil could be would of many strands in parallel of 0.0075-in.-diam Nb-Zr wire coated with an insulating layer of Teflon of 0.00025-in. thickness. A 0.001-in.-thick sheet of copper would be placed between layers to shield against voltage gradients due to changing magnetic flux during energy withdrawal. The ends of each winding are connected to the contacts of the switch shown in the figure such that each winding is either short-circuited through the switch or connected to the external load. As the switch actuator is advanced upward, the shorting bar breaks contact with the lower coil circuit and the coil current is forced to flow through the external load. If the actuator is driven at a linear rate, the output current to the load would build up to a d.c. level in the manner indicated in Fig. 3. All switch components must be made of superconducting materials. As the magnetic field strength in the neighborhood of the switch would be quite low compared to that in the coil, any of a wide variety of "soft" superconductors with low magnetic field limits could be used for the switch. The feasibility of a mechanical superconducting switch remains to be demonstrated. The coil, switch, and output leads are bathed in liquid helium in order to maintain superconductivity. The entire device is enclosed in a stainless steel case having a thermal liner of evacuated superinsulation to inhibit helium loss.

The device initially could be energized by moving the switch to its lowermost position, where the charging contacts are in the circuit of the lowermost coil. A d.c. power source capable of supplying a current greater than the one desired

† Patent applied for.

in that winding is then attached to the input power terminals. When the current in the first winding reaches the desired value, the actuator is advanced so that the current through the first winding now passes through the shorting bar and the next winding is being energized. In this way the entire device can be energized with a low output power supply.

The arrangement described permits conversion of an exponentially decreasing current into a d.c. output with a long period sawtooth ripple, which would not be objectional in many applications. The magnitude of the current ripple would correspond to the current capacity of a single winding. The power output of the device would be determined by the rate of travel of the actuator.

Optimization of Energy Storage

The optimization of the system is based on determining the maximum energy storage capability per unit volume of coil material for a given maximum magnetic field B_{max} at the inner radius of the coil, and the corresponding current density J as determined from Fig. 1. In the following analysis mks units are used throughout. The energy stored in a coil of inductance L, carrying current I is

$$E = \frac{1}{2} LI^2$$

This may be expressed as energy per unit volume:

$$\frac{E}{V} = \frac{1}{2} \frac{L}{V} I^2 \tag{1}$$

Assuming a coil of rectangular cross section, the coil dimensions a, b, and c are defined in Fig. 4. If the number of turns n carrying the current I is known, the effective cross-sectional area required for each turn is

$$A_o = \frac{bc}{n} \tag{2}$$

The standard formula for the inductance of a solenoid or coil of rectangular cross section is[4]

$$L = 3.948 \frac{a^2 n^2}{b} \left[K - 0.318 \frac{c}{a} (0.693 + B_s)\right] \cdot 10^{-6} \text{ h} \tag{3}$$

where K and B_s, tabulated in the reference, are shown graphically in Fig. 5 and 6, respectively. The relationship between the magnetic field generated at the center of the coil B_o, and the current through the coil is as follows:[5]

$$B_o = \frac{nI}{2b} \frac{b}{\sqrt{a^2 + 0.25\, b^2}} 4\,\pi \cdot 10^{-7} \text{ weber/m}^2 \qquad (4)$$

With the help of Eq. (4) and (2), Eq. (1) can be rewritten as

$$\frac{E}{V} = \frac{B_o^{\,2}}{8\pi^2} \left(\frac{A_o^{\,2}}{a^2} \; \frac{L}{V} \right) \left(\frac{a}{c} \right)^2 \left[\left(\frac{a}{b} \right)^2 + 0.25 \right] \cdot 10^{14} \text{ joule/m}^3 \qquad (5)$$

which gives the energy density as a function of B_o. It is not the field at the center of the coil which limits the attainable current density within the superconducting wires, but the field adjacent to the wires. If the current density in the wires is assumed constant throughout the entire coil, the critical current density, as defined by Fig. 1, will be determined by the maximum field existing within the coil windings. For the assumed coil shape, this point of maximum field B_{max} will occur at the inner radius of the coil midway between the ends. For convenience the ratio of B_{max} to B_o will be called k. A function of b/a and c/a, k has been determined by IBM computer analysis in Ref. 6. A plot of k is given in Fig. 7. Incorporating k into Eq. (5) gives

$$\frac{E}{V} = \frac{B_{max}^{\;\;2}}{8\pi^2\, k^2} \left(\frac{A_o^{\,2}}{a^2} \; \frac{L}{V} \right) \left(\frac{a}{c} \right)^2 \left[\left(\frac{a}{b} \right)^2 + 0.25 \right] \cdot 10^{14} \qquad (6)$$

This gives the energy density as a function of B_{max}, but since J and B_{max} are interrelated, it is necessary to formulate an expression for energy density in terms of J also, in order to solve for the coil dimensions required to give a specified energy density. Here I may be expressed in terms of J by

$$I = JA$$

where A is the superconducting cross-sectional area of a single conductor. Substitution of the foregoing into Eq. (1) gives

$$\frac{1}{a^2} \frac{E}{V} = \frac{1}{2} \left(\frac{A}{A_o} \right)^2 \left(\frac{A_o^{\,2}}{a^2} \; \frac{L}{V} \right) \cdot J^2 \qquad (7)$$

For simplification let

$$C = \left(\frac{A_o^2}{a^2} \quad \frac{L}{V}\right) \cdot 10^6$$

Defining the reference volume V as that occupied by the coil material,

$$V = 2\pi abc \tag{8}$$

and making use of Eqs. (2) and (3),

$$C = 0.628 \frac{c}{a} \left[K - 0.318 \frac{c}{a} (0.693 + B_s)\right] \tag{9}$$

Therefore, C is seen to be dependent only on geometric ratios. One additional quantity D will be defined:

$$D = C\left(\frac{a}{c}\right)^2 \left[\left(\frac{a}{b}\right)^2 + .25\right] \tag{10}$$

Equations (6) and (7) may now be rewritten

$$\frac{E}{V} = \frac{B_{max}^2}{8\pi^2} \cdot \frac{D}{k^2} \cdot 10^8 \tag{11}$$

$$\frac{1}{a^2} \frac{E}{V} = \frac{J^2}{2}\left(\frac{A}{A_o}\right)^2 \cdot C \cdot 10^{-6} \tag{12}$$

For a given B_{max} and J, as determined from Fig. 1, and selected values of the ratios b/a and c/a the coil radius can be determined from solution of Eqs. (11) and (12). With the help of Eq. (8) the total stored energy and the weight of the coil are determinable.

For the purpose of graphical presentation of the characteristics and capabilities of inductive energy storage devices, it is first necessary to select a value for A/A_o. For the configuration described earlier using the 0.0075-in.-diam Nb-Zr wire, the value of this ratio is 0.613, whereas for the Nb_3Sn wire with a superconducting core diameter half the wire diameter, a 0.0075-in.-diam wire gives an $A/A_o = 0.153$. The determination of the stored energy, coil radius, and coil weight for selected values of B_{max} and J are given in Figs. 8 to 10, for a b/a of 1.0. The identity of the curves is as follows:

Curve A: Nb-Zr wire. B = 3.0 weber/m^2; J = 16.0 x 10^8 amp/m^2
Curve B: Nb-Zr wire. B = 8.0 weber/m^2; J = 2.0 x 10^8 amp/m^2
Curve C: Nb_3Sn wire. B = 12.5 weber/m^2; J = 10.0 x 10^8 amp/m^2

A single curve represents a range of values of c/a. The smaller values of c/a give the higher values of energy density.

The choice of b/a = 1.0 is a compromise between a long coil, which would give a slightly higher energy density, and a short coil, which would give a more compact unit with a lower insulated casing weight and a smaller liquid helium requirement. The short coil gives a lower energy density because of its high k, as shown in Fig. 7.

It is evident from the curves that coils of large radii are required to give high energy densities. Nb_3Sn coils perform much better than Nb-Zr coils in this respect. This behavior is attributed to the higher achievable magnetic fields with Nb_3Sn coils. Such high field coils are not satisfactory for low energy storage requirements, for example 10^6 joule, in which instance a lower magnetic field gives a higher energy density, as is indicated by curve A. In order to achieve high energy densities, high field coils must be of large radii.

Illustrative Examples

To date solenoids have been built by various research laboratories which generate fields up to 6.0 weber/m^2. These solenoids, intended for producing high magnetic fields, are very poor energy storage devices, however. To illustrate by an example, one laboratory has built a solenoid which is 5.0 cm in length and 5.6 cm in diameter, and which develops a field of 5.9 weber/m^2 in its 0.5 cm central tube. This solenoid operates at a current density of 3.75×10^8 amp/m^2, which is very close to the measured limits of 4.0×10^8 amp/m^2 at 6.0 weber/m^2 shown in Fig. 1. This solenoid will store 275 joule at an energy storage density of 2.25×10^6 joule/m^3. By using the same length of wire, but increasing the coil radius to its optimum value for a reduced field of 3.0 weber/m^2, a corresponding current density of 1.6×10^9 amp/m^2, and a b/a = 1.0, the stored energy jumps to 4000 joule. The mean coil diameter is now 14.2 cm.

To take an example involving a large amount of stored energy, assume that 1.5×10^8 joule of energy are required to provide 2.5 Mw of power for a period of one minute. Using curve C of Fig. 8, the required storage density is seen to be 3.1×10^6 joule/m^3. Reference to Fig. 9 and 10 reveals the mean coil radius to be 0.922 m and the coil weight to be 3040 kg. Assume that the coil and associated components are to be enclosed in a double-walled toroidal container constructed of 1/16 in. stainless steel with a 3-cm-thick, evacuated insulating layer of NRC-2[7] superinsulation. This insulation has a conductivity of 0.3 μw/cm-°K. Such a structure with its helium refrigerant would weigh 376 kg. If a TiO_2-coated container in an earth orbit were to attain a maximum surface temperature of 150°K, the total heat influx from the surface through the insulator to the liquid helium would then be 2.1w. A 100-kg helium refrigerator[8] would be required to absorb this heat. Therefore the minimum total system weight would be 3516 kg, and the effective specific energy storage would be 4.26×10^4 joule/kg. The weight of the 75-w auxiliary power source for the refrigerator has been neglected.

Comparison With Other Power Sources

The great advantage of inductive storage of energy is the capability of supplying this energy to a load at a very high rate. The size of the storage coil affects only the total energy available and not the power output. Devices such as the fuel cell are severely limited in their power output capabilities. Batteries are considerably superior to fuel cells. One of the best for high power applications is the zinc-silver oxide batter, which can supply an output of 0.66 kw/kg at a specific energy of 4.9×10^4 joule/kg,[9] which is comparable to the example of inductive storage device given in the foregoing. This, however, is the limit of the battery's power output, whereas the inductance coil could supply 15 Mw for 10 sec if desired, giving a specific power exceeding 4 kw/kg. No other power source is at all competitive in this respect.

Conclusions

The designs given are based on the present state of the technology of superconducting materials. Great advances are yet expected in improving the properties of hard superconductors. Still using the modest design criteria presented here, the superconducting coil has been shown to be superior for high power output to any other method of energy storage. Its obvious use is in a space power system.

A small unit could be charged by solar cells or other means at a slow rate, and then could supply a short burst of power in the megawatt range as required for high power transmitter operation, laser beam communications, or perhaps for high thrust impulses from an arc jet or other electric propulsor at perigee and apogee to reduce inter-orbit transit time.[10] A large spacecraft or satellite station containing a low power, nuclear-electric power system would have a ready means of recharging a large inductive storage unit, which could provide very high power on a low duty cycle for the life of the reactor, thereby possibly eliminating the need for a heavy, high power nuclear-electric system.

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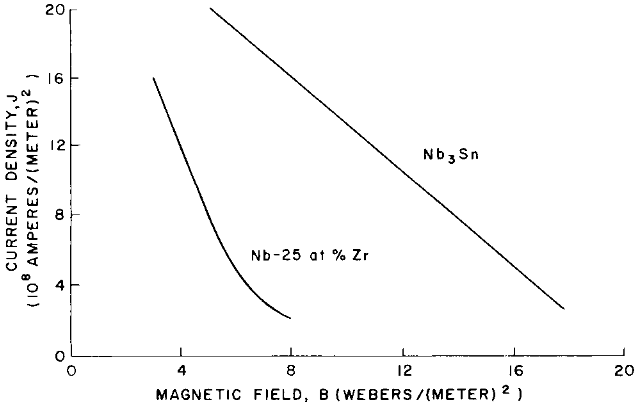

Fig. 1 Critical current density vs magnetic field at 4.2°K for two types of superconductors

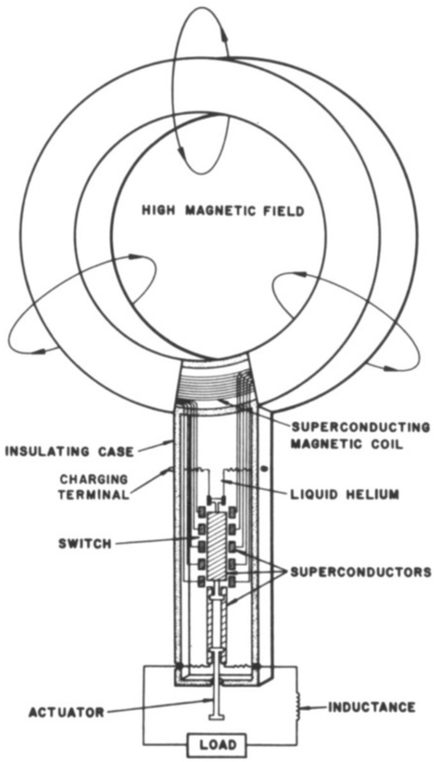

Fig. 2 Configuration of a superconducting inductive energy storage device capable of providing a d.c. output current

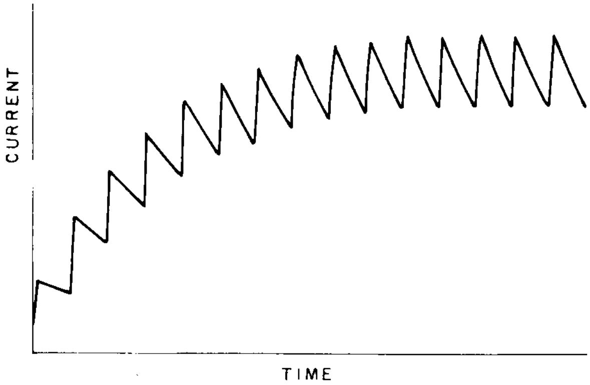

Fig. 3 Characteristic current output waveform

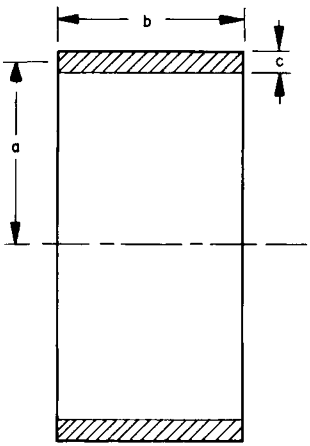

Fig. 4 Cross-sectional geometry of coil

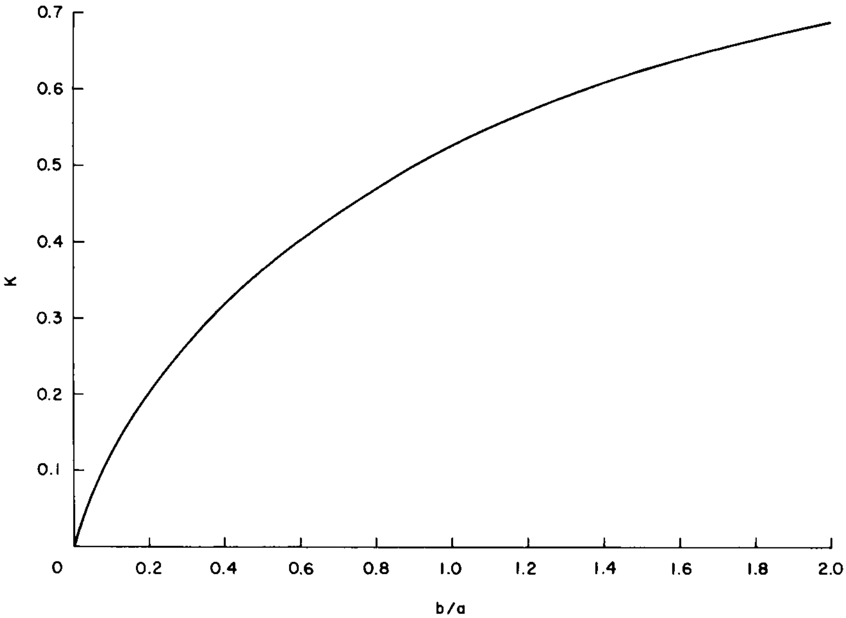

Fig. 5 Inductance factor K for use in Eq. (9)

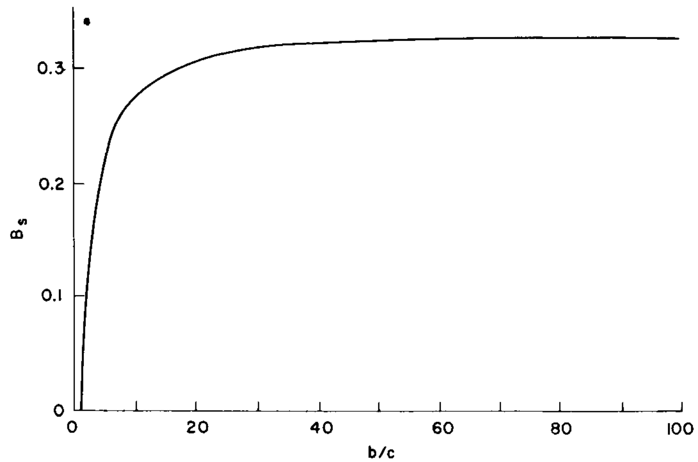

Fig. 6 Inductance correction factor B_s for use in Eq. (9)

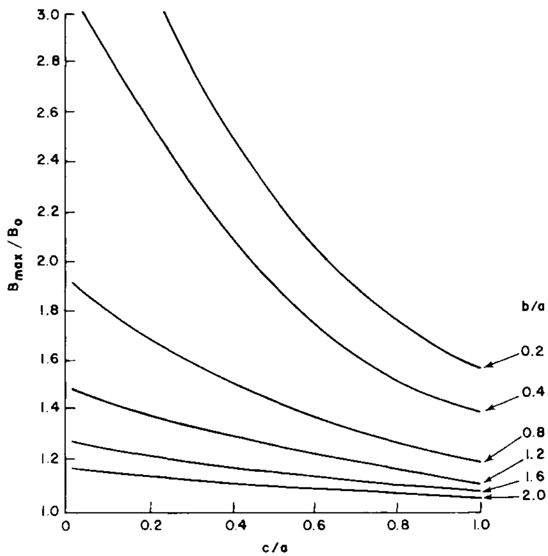

Fig. 7 Effect of coil geometry on the ratio of the maximum magnetic field within the coil to the field at the midpoint of the coil axis

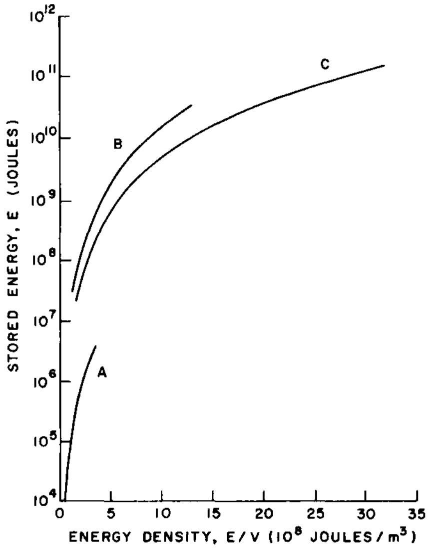

Fig. 8 Total energy capacity of coil for Nb-Zr at 3.0 weber/m^2 in curve A, 8.0 weber/m^2 in curve B, and for Nb_3Sn at 12.5 weber/m^2 in curve C

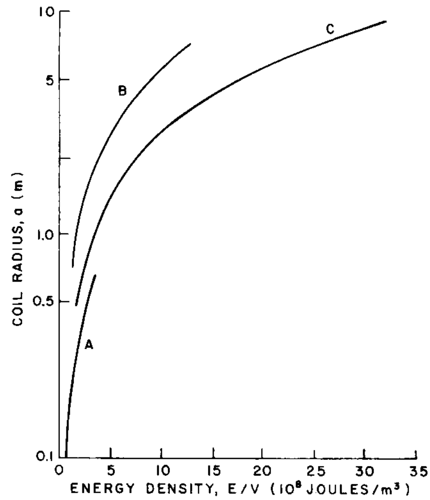

Fig. 9 Mean radius of coil for Nb-Zr at 3.0 weber/m^2 in curve A, 8.0 weber/m^2 in curve B, and for Nb_3Sn at 12.5 weber/m^2 in curve C

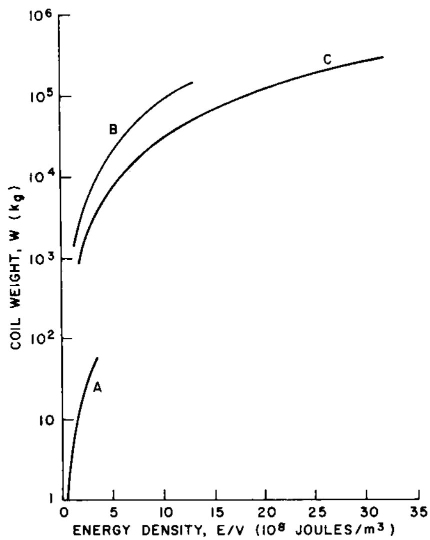

Fig. 10 Weight of coil windings for Nb-Zr at 3.0 weber/m^2 in curve A, 8.0 weber/m^2 in curve B, and for Nb_3Sn at 12.5 weber/m^2 in curve C